The Effects of Reed-Solomon Code Shortening on the Performance of Coded Telemetry Systems

L. J. Deutsch
Communications Systems Research Section

In the proposed NASA/ESA telemetry/coding standard, a (255, 223) Reed-Solomon code is concatenated with an inner (7, 1/2) convolutional code. Under some circumstances, it would be desirable to use a shorter outer code word length. For example, the format of the data coming from science instruments on board a spacecraft may lend itself naturally to a word length of 200 symbols rather than 223. To accommodate such code word lengths, the Reed-Solomon code can be shortened to an (N, N-32) code where N can be any integer between 33 and 255. Shortening the code, however, changes its performance. On one hand, the amount of redundancy per information symbol increases. This would, by itself, imply that performance would improve. However, because of this increased redundancy, the amount of energy per information symbol is decreased by code shortening. The overall effect is to degrade the performance of the code. This report develops the theory of Reed-Solomon code shortening in general and quantifies the degradation due to shortening in the context of concatenated coding. It is shown that in the NASA/ESA concatenated system, significant degradations (greater than 0.1 dB at a bit error rate of 10^{-6}) occur only when $N < 180$.

I. Introduction

All planned NASA and European Space Agency (ESA) deep space missions are expected to have the capability of using a concatenated Reed-Solomon/convolutional coding scheme for downlink telemetry. In fact, this coding system is a proposed NASA/ESA standard (Ref. 1). The inner code is a (7, 1/2) convolutional code. This is the same code that is currently used by the Voyager spacecraft. The outer code is a (255, 223) Reed-Solomon code. The proposed standard code is slightly different than that used by Voyager in that a different representation is used to represent the eight-bit Reed-Solomon symbols. The two Reed-Solomon codes, however, share the same code length parameters and hence have identical performances. Many studies have been performed to determine the performance of this concatenated coding system under various conditions (Refs. 1,2,3, and 4).

Since the Reed-Solomon code words consist of 223 eight bit information symbols, 1,784 bits are required from the spacecraft's data system to encode each code word. Some of these bits will typically be frame headers that contain identification, timing, and synchronization information. The remainder of the bits are data from various scientific instruments. There could conceivably be cases in which the spacecraft instruments produce data in a form that is more amenable to being packed into a smaller number of bits. In fact, it is even possible to imagine scenarios for which the number of bits that form such an information packet might vary with time. Under such conditions, it is desirable for the Reed-Solomon encoder to be able to process fewer than 1,784 bits at a time.

Fortunately, this is possible by adapting the (255, 223) code for use as a (N, N-32) code for $N \leq 255$. One way of accomplishing this is illustrated in Fig. 1. Suppose that $M = N$-32 information symbols are generated by the spacecraft data system and that $M < 223$. To these symbols, the Reed-Solomon processor appends 223 - M additional symbols, all of which happen to be zeroes. Since there are now 223 symbols, encoding can take place. A code word consisting of 255 symbols is generated. Since all Reed-Solomon codes that are planned for use in space missions are systematic, the information symbols are themselves a portion of the code word – in this case they are the first 223 symbols. The zeroes that were added for the purposes of encoding are now stripped away and the rest of the code word is sent to the convolutional encoder. After Viterbi decoding on the ground, the zeroes are once again added to the code word. This allows it to be decoded. Following Reed-Solomon decoding, the zeroes are finally stripped away to reveal the "original" information sequence.

The above process is an example of Reed-Solomon code shortening. The effect is to use a (255, 223) encoder and decoder, to implement an (N, N-32) code. In general, the fixed sequence that is added to the input information sequence can be of any form. It is called the "fill sequence." If, as in the above example, it is the all zero sequence, then the process is sometimes referred to as "virtual zero fill." The fill sequence can be merged with the information sequence in any way – not just at the end as in the example. It is crucial, however, that the fill sequence be added in the same way at the decoding end of the system.

There are two phenomena that occur in Reed-Solomon code shortening that affect the overall performance of the system. The first is that the code rate is changed with shortening. The code rate of the original Reed-Solomon code is 223/255. The code rate of a length N shortened RS code is $(N - 32)/N$. Since

$$(N - 32)/N < 223/255$$

for all $N < 255$, shortening reduces the code rate. If all other aspects could be held constant, this might imply improved performance. However, a second effect of code shortening is to reduce the energy that is expended for each Reed-Solomon code word in the transmitter. This tends to degrade the overall performance. In fact, the second effect is greater than the first. Reed-Solomon code shortening does degrade the performance of the concatenated coding system.

In the following sections of this report, a theory of Reed-Solomon code shortening is developed. The performance of a concatenated (7, 1/2) convolutional/$(N - 32, N)$ Reed-Solomon system is also calculated. It is shown that the degradation is actually quite small. Only if the Reed-Solomon code word length is shortened to $N = 180$ symbols or less will there be a 0.1 dB loss at a concatenated decoded bit error rate of 10^{-6}.

II. The Theory of Reed-Solomon Code Shortening

Throughout the remainder of this report, it is assumed that the coding system is the proposed standard concatenated system shown in Fig. 2.

Suppose that the energy in each channel symbol (the digital entities that are output from the convolutional decoder) is E_s. Then the symbol energy to noise spectral density ratio is E_s/N_0. Since the rate of the convolutional inner code is 1/2, the signal to noise ratio for the bits that are input to the convolutional encoder is given by

$$\frac{E_V}{N_0} = 2\frac{E_s}{N_0}$$

Now suppose that the (255, 223) Reed-Solomon code is shortened to length N ($32 < N \leq 255$). This means that the Reed-Solomon encoder assumes that the $N - 32$ input information Reed-Solomon (RS) symbols are merged with a fill sequence of length 255 - N RS symbols to form an input sequence of length 223 RS symbols. The encoder can then generate the 32 parity check symbols that are appended to the input sequence to form a length N code word. The rate of the shortened code is

$$R_N = \frac{N-32}{N}$$

This implies that the information bit (bits input to RS encoder) signal to noise ratio is

$$\frac{E_b}{N_0} = \frac{N}{N-32}\frac{E_V}{N_0} = \frac{2N}{N-32}\frac{E_s}{N_0} \tag{1}$$

The performance of the Viterbi decoded inner code can be expressed in terms of E_V/N_0. Let

$$p = f\left(\frac{E_V}{N_0}\right) \tag{2}$$

and

$$\pi = g\left(\frac{E_V}{N_0}\right) \tag{3}$$

represent the Viterbi-decoded bit error rate and the error rate for sets of eight consecutive bits respectively. The quantity π is also the input RS symbol error rate for the Reed-Solomon decoder. If the Reed-Solomon code is infinitely interleaved, then the overall bit error rate of the concatenated code is

$$p_{bit} = \frac{p}{\pi} \sum_{j=17}^{N} \left(\frac{j}{N}\right)\binom{N}{j}\pi^j (1-\pi)^{N-j} \tag{4}$$

This theory can be applied to any concatenated coding system where the outer code is a shortened (255, 223) Reed-Solomon code. This is done by appropriately defining p and π to be the bit error rate and the error rate for sets of eight bits respectively for the inner code. One special case is when the inner code is non-existent. In this case, the coding system consists only of the Reed-Solomon code. The bit error rate, p, of the "inner code" is just the bit error rate of uncoded transmission.

If the channel is memoryless, then

$$\pi = 1 - (1-p)^8$$

III. Numerical Results

The theory developed in Section II was used to evaluate the performance of concatenated coding with Reed-Solomon code shortening. The values of p and π that define the performance of the (7, 1/2) Viterbi-decoded convolutional code came from software simulations that were reported in Ref. 3. These functions are shown graphically in Fig. 3.

The Reed-Solomon decoded bit error probability was then computed as a function of E_b/N_0 [Eqs. (1) - (4)]. Fig. 4 shows the results of these calculations for several values of the shortened code length N. The full code case ($N = 255$) is included for comparison. It should be noted that these results assume that there are no degradations from the analogue parts of the telemetry receiving system and that both convolutional and Reed-Solomon code synchronization are maintained perfectly. Also, infinite interleaving of Reed-Solomon symbols is assumed. It is true, however, in the full code length case, that the difference in performance between an interleaving depth of five and one of infinity is negligible.

The loss due to the code shortening is the additional E_b/N_0 that must be added to the signal to make the shortened code's performance equal to that of the full length code. For a fixed overall bit error rate, this degradation is just the horizontal distance between the corresponding curves on the graph measured at that error rate. Graphs of this loss for overall bit error rates of 10^{-4}, 10^{-5}, and 10^{-6} are shown in Fig. 5.

IV. Conclusions

The results of the computations performed in Section III show that the degradation due to Reed-Solomon code shortening in the concatenated coding system is small for moderate amounts of shortening. Even at a bit error rate of 10^{-6}, a code word length of less than 180 must be used before a loss of 0.1 dB exists. In fact, the loss due to shortening does not vary much with the bit error rate (at least for probabilities less than 10^{-4}). This is because the Reed-Solomon performance curve is almost a perfect exponential function in this region.

It is not known at this time how degradations in the receiver, subcarrier tracking loop, and symbol synchronizer may effect these results. It is also not known how node synchronization losses in Viterbi decoder (Ref. 5) or frame synchronization losses in the Reed-Solomon decoder (Ref. 6) will affect the performance of shortened codes. However, it seems that a small amount of shortening can be accommodated with only a negligible loss in overall system performance.

References

1. Layland, J. W., "Information Systems: Performance of Short Constraint Length Convolutional Codes and a Heuristic Code-Construction Algorithm," *Space Program Summary 37-64*, Vol. II, Jet Propulsion Laboratory, Pasadena, Calif., August, 1970.

2. Odenwalder, J. P., "Concatenated Reed-Solomon/Viterbi Channel Coding for Advanced Planetary Missions: Analysis, Simulations, and Tests," submitted to the Jet Propulsion Laboratory by Linkabit Corporation., San Diego, Calif., Contract No. 953866, December, 1974.

3. Miller, R. L., Deutsch, L. J., and Butman, S. A., *On the Error Statistics of Viterbi Decoding and the Performance of Concatenated Codes*, Publication 81-9, Jet Propulsion Laboratory, Pasadena, Calif., September, 1981.

4. Butman, S. A., Deutsch, L. J., Miller, R. L., "New Results on Antenna Arraying: Part One," *TDA Progress Report 42-62*, Jet Propulsion Laboratory, Pasadena, Calif., January and February, 1981.

5. Deutsch, L. J., and Miller, R. L., "Viterbi Decoder Node Synchronization Losses in the Reed-Solomon/Viterbi Concatenated Channel," *TDA Progress Report 42-71*, Jet Propulsion Laboratory, Pasadena, Calif., July-September, 1982.

6. Swanson, L., *A Comparison of Frame Synchronization Methods*, Publication 82-100, Jet Propulsion Laboratory, Pasadena, Calif., December, 1982.

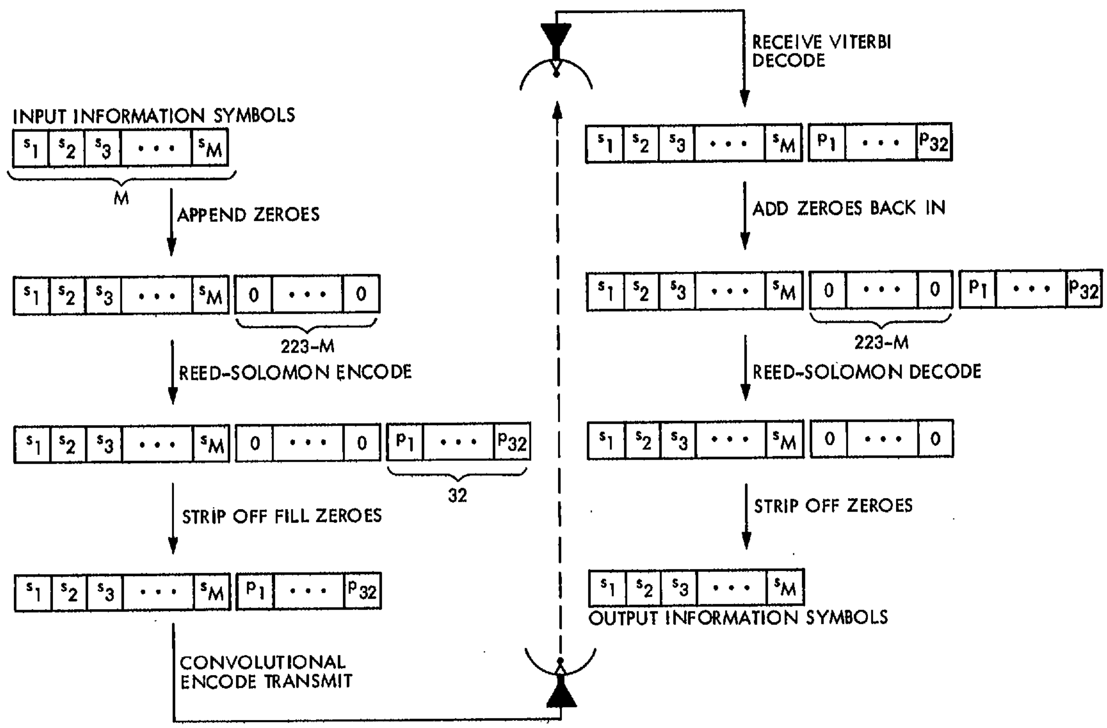

Fig. 1. An example of concatenated coding with Reed-Solomon code shortening

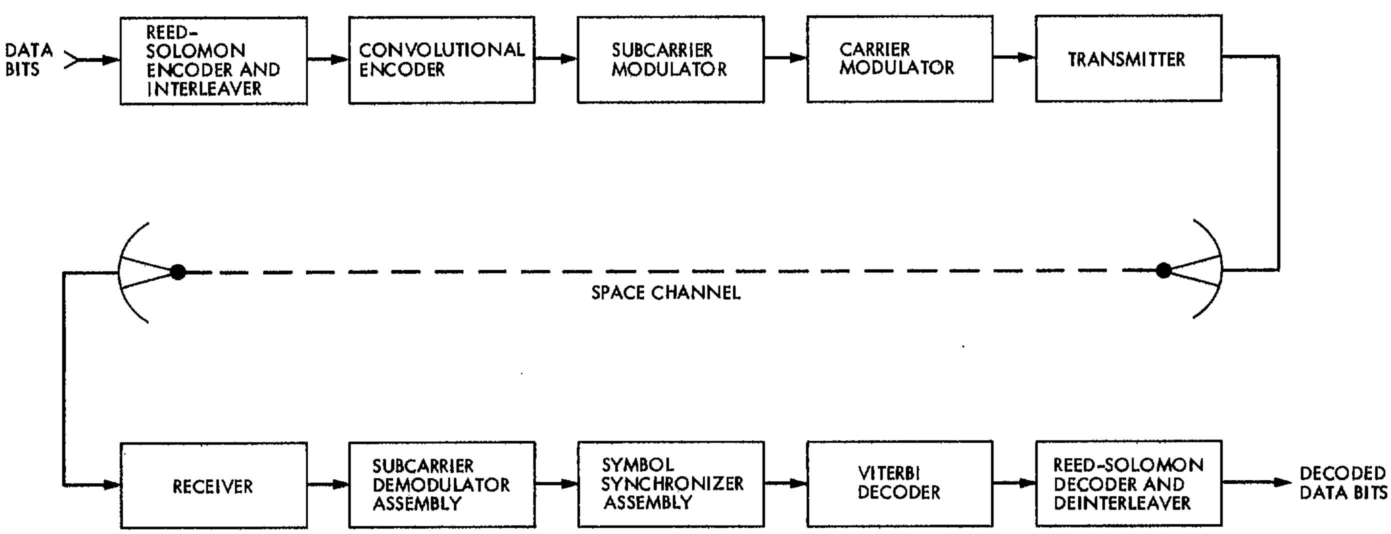

Fig. 2. Concatenated coding system block diagram

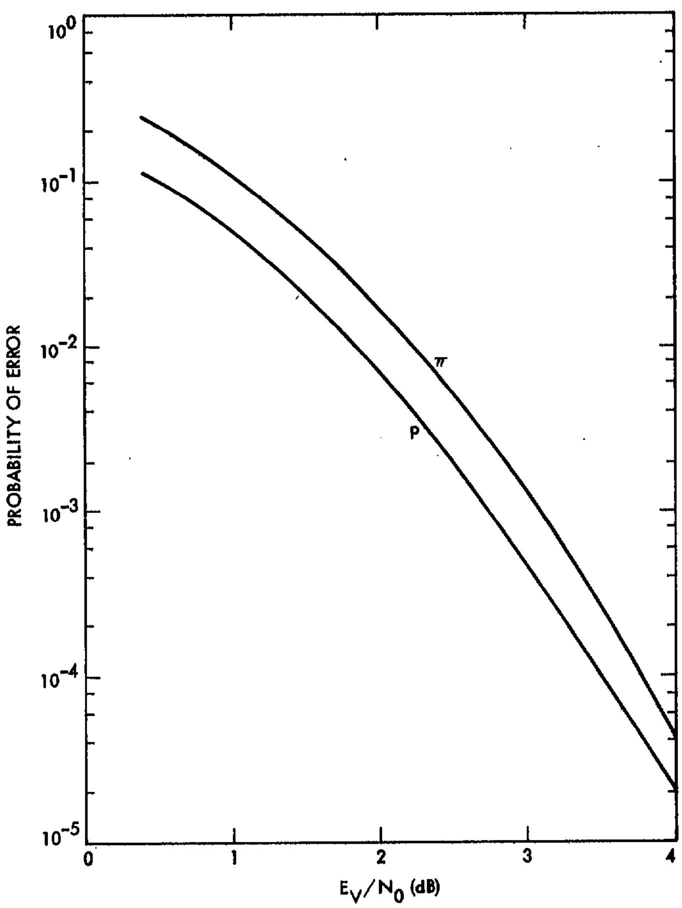

Fig. 3. Bit (p) and symbol (π) error rate performance of (7, 1/2) Viterbi decoder

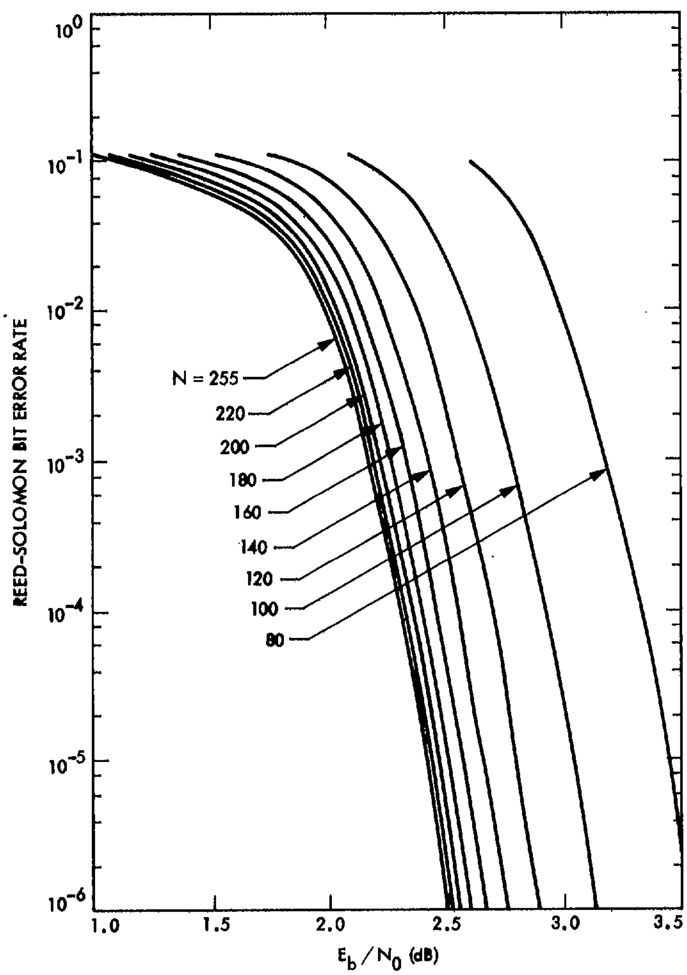

Fig. 4. Reed-Solomon bit error rate performance as a function of shortened code word length N

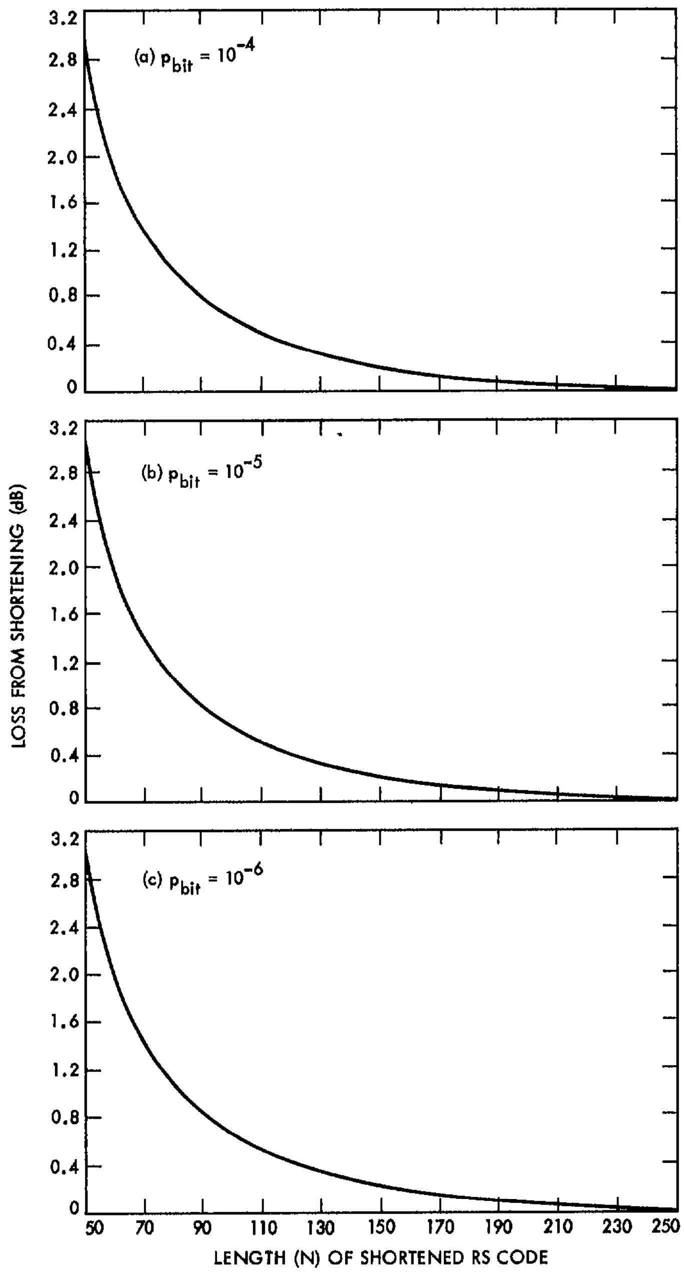

Fig. 5. Loss due to Reed-Solomon code word shortening